Ad Soyad:

Numara:

14.06.2019

MAT 324 MATRİSLER TEORİSİ BÜTÜNLEME SINAVI SORULARI

1. Bir ters hermit matrisinin köşegen elemanlarının sıfır yada sanal sayı olduğunu gösteriniz.

2. $A=\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 2 & 1 & 4 \end{bmatrix}$ matrisinin tersini bulunuz.

3. $$\begin{cases} 4x_1+3x_2-9x_1+x_4=1 \\ -x_1+2x_2-13x_3+3x_4=3 \\ 3x_1-x_2+8x_3-2x_4=-2 \end{cases}$$

sistemini elemanter işlemler yardımıyla çözünüz.

4. $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ matrisinin determinantını determinant tanımını kullanarak hesaplayınız.

Not: Her soru eşit puanlıdır.

BAŞARILAR.

Süre: 75 dk

CEVAPLAR

1) A ters hermit matris olsun. $A = [a_{ij}]$

A ters hermit $\Rightarrow (\bar{A})^t = -A$ dır.

$\Rightarrow [\overline{a_{ij}}]^t = -[a_{ij}]$

$[\overline{a_{ji}}] = -[a_{ij}] \Rightarrow [\overline{a_{ji}}] = [-a_{ij}]$

$\Rightarrow \overline{a_{ji}} = -a_{ij}$

A'nın köşegen elemanları için $i=j$ dir.

$\Rightarrow \overline{a_{ii}} = -a_{ii} \Rightarrow \overline{a_{ii}} + a_{ii} = 0$

" $z = a + ib \Rightarrow \bar{z} = a - ib$

$z + \bar{z} = 2a = 0 \Rightarrow a = 0 \Rightarrow \boxed{z = ib}$

b sıfır olabilir.

$\Rightarrow a_{ii} = 0$ veya a_{ii} sanaldır.

2)

$$[A:I] = \begin{array}{c} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 2 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \end{array} \begin{array}{l} -\alpha_1 \\ -\alpha_2 \\ -\alpha_3 \end{array}$$

$E_1: \alpha_2 \to \alpha_2 - \alpha_1$

$\alpha_3 \to \alpha_3 - 2\alpha_1$

$E_2: \alpha_1 \to \alpha_1 - 2\alpha_2$

$\alpha_3 \to \alpha_3 + 3\alpha_2$

$$[A:I] \overset{E_1}{\approx} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & -3 & 2 & -2 & 0 & 1 \end{array}\right] \overset{E_2}{\approx} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 3 & -2 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 2 & -5 & 3 & 1 \end{array}\right]$$

$E_3: \alpha_3 \to \frac{1}{2}\alpha_3$

$$\overset{E_3}{\approx} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 3 & -2 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{5}{2} & \frac{3}{2} & \frac{1}{2} \end{array}\right] \overset{E_4}{\approx} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{11}{2} & -\frac{7}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{5}{2} & \frac{3}{2} & \frac{1}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 11/2 & -7/2 & -1/2 \\ -1 & 1 & 0 \\ -5/2 & 3/2 & 1/2 \end{bmatrix}$$

3)
$$\begin{array}{c} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{array}\left[\begin{array}{cccc|c} -4 & 3 & -9 & 1 & 1 \\ -1 & 2 & -13 & 3 & 3 \\ -3 & -1 & 8 & -2 & -2 \end{array}\right] \overset{\mathcal{E}_1}{\sim} \left[\begin{array}{cccc|c} 1 & 4 & -17 & 3 & 3 \\ -1 & 2 & -13 & 3 & 3 \\ 3 & -1 & 8 & -2 & -2 \end{array}\right]$$

$\mathcal{E}_1 : \alpha_1 \to \alpha_1 - \alpha_3$ $\qquad$ $\mathcal{E}_2 : \begin{array}{l} \alpha_2 \to \alpha_2 + \alpha_1 \\ \alpha_3 \to \alpha_3 - 3\alpha_1 \end{array}$ $\qquad$ $\mathcal{E}_3 : \alpha_2 \to \frac{\alpha_2}{6}$

$$\sim \left[\begin{array}{cccc|c} 1 & 4 & -17 & 3 & 3 \\ 0 & 6 & -30 & 6 & 6 \\ 0 & -13 & 59 & -11 & -11 \end{array}\right] \overset{\mathcal{E}_3}{\sim} \left[\begin{array}{cccc|c} 1 & 4 & -17 & 3 & 3 \\ 0 & 1 & -5 & 1 & 1 \\ 0 & -13 & 59 & -11 & -11 \end{array}\right]$$

$$\overset{\mathcal{E}_4}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 3 & -1 & -1 \\ 0 & 1 & -5 & 1 & 1 \\ 0 & 0 & -6 & 2 & 2 \end{array}\right] \overset{\mathcal{E}_5}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 3 & -1 & 1 \\ 0 & 1 & -5 & 1 & 1 \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} \end{array}\right]$$

$$\overset{\mathcal{E}_6}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{2}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 0 \\ x_2 - \frac{2}{3}x_4 = -\frac{2}{3} \\ x_3 - \frac{x_4}{3} = -\frac{1}{3} \end{array}$$

$x_4 = t$ τότε $x_1 = 0$ $\quad x_2 = -\frac{2}{3} + \frac{2}{3}t$, $\quad x_3 = -\frac{1}{3} + \frac{t}{3}$

4)
$$A = \left[\begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{array}\right] \begin{array}{l} -\alpha_1 \\ -\alpha_2 \\ -\alpha_3 \end{array}$$

$$\det A = f(\alpha_1, \alpha_2, \alpha_3) = \sum_{\sigma \in S_3} s(\sigma)\, a_{\sigma(1)1}\, a_{\sigma(2)2}\, a_{\sigma(3)3}$$

$$\left\{ \sigma_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \right.$$
$$\left. \sigma_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \sigma_5 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \sigma_6 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \right\}$$

$$\begin{aligned}\det A = &\underbrace{S(\sigma_1)}_{+1} a_{\sigma_1(1)1} a_{\sigma_1(2)2} a_{\sigma_1(3)3} \\ &+ \underbrace{S(\sigma_2)}_{-1} a_{\sigma_2(1)1} a_{\sigma_2(2)2} a_{\sigma_2(3)3} \\ &+ \underbrace{S(\sigma_3)}_{+1} a_{\sigma_3(1)1} a_{\sigma_3(2)2} a_{\sigma_3(3)3} \\ &+ \underbrace{S(\sigma_4)}_{+1} a_{\sigma_4(1)1} a_{\sigma_4(2)2} a_{\sigma_4(3)3} \\ &+ \underbrace{S(\sigma_5)}_{-1} a_{\sigma_5(1)1} a_{\sigma_5(2)2} a_{\sigma_5(3)3} \\ &+ \underbrace{S(\sigma_6)}_{-1} a_{\sigma_6(1)1} a_{\sigma_6(2)2} a_{\sigma_6(3)3} \\ = &\, a_{11}a_{22}a_{33} - a_{12}a_{21}a_{33} + a_{13}a_{21}a_{32} \\ &+ a_{12}a_{23}a_{31} - a_{11}a_{23}a_{32} - a_{13}a_{22}a_{31}\end{aligned}$$